Section A
All Questions are compulsory, No internal choice is provided in this section

Question 1.
The value of -70 mod 13 is
(a) 5
(b) -5
(c) 8
(d) -8
Solution:
(c) To find-70 mod 13
Let us divide -70 by 13

$$13)-70(-6$$
$$\underline{-78}$$
$$8$$

So, $-70 \bmod 13 = 8$

Question 2.
If x+1x+2 ≥ 1, then
(a) x ∈ [-∞, 2]
(b) x ∈ (-∞, -2)
(c) x ∈ (-∞, 2]
(d) x ∈ [-∞, 2)
Solution:
(b) Given, $\frac{x+1}{x+2} \geq 1$

$\Rightarrow \frac{x+1}{x+2} - 1 \geq 0$

$\Rightarrow \frac{x+1-x-2}{x+2} \geq 0$

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$\Rightarrow \frac{-1}{x+2} \geq 0$

$\Rightarrow x+2 < 0$

$\left[\because \frac{a}{b} > 0 \text{ and } a < 0 \Rightarrow b < 0\right]$

$\Rightarrow x < -2 \Rightarrow x \in (-\infty, -2)$

Question 3.
Which of the following is a statistic?
(a) μ
(b) $\overline{x}$
(c) σ²
(d) None of these
Solution:
(b) $\overline{x}$ is a statistic.
Question 4.
In one sample test, the estimation for population mean is

(a) $\frac{\bar{x}-\mu}{\frac{S}{\sqrt{n}}}$ (b) $\frac{\bar{x}-\mu}{\frac{S}{n}}$ (c) $\frac{\bar{x}-\mu}{\frac{S^2}{n}}$ (d) None of these LearnCBSE.in

Solution:
(a) The estimation for population mean is
x¯¯−μSn√
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Question 5.
A man can row 6 km/h in still water. It takes him twice as long to row up as to row down the river. Then, the rate of the stream is
(a) 2 km/h
(b) 4 km/h
(c) 6 km/h
(d) 8 km/h
Solution:
(a) Let man's speed in upstream = x km/h
Let man's speed in downstream = 2x km/h
Hence, man's speed in still water
= 12(speed in upstream + speed in dowenstream)
= 12(x + 2x) = 3x2km/h
∴ 6 = 3x2 ⇒ x = 4 km/h in upstream
and downstream = 2 × 4 = 8 km/h
Speed of stream = 12 (8 – 4) = 42 = 2 km/h
Question 6.
If random variable X represents the number of heads when a coin is tossed twice, then mathematical expectation of X is
(a) 0
(b) 14
(c) 12
(d) 1
Solution:
(d) A coin is tossed twice
x represent number of heads
∴ Probability distribution

| X | 0 | 1 | 2 |
|---|---|---|---|
| P (X) | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ |

Mathematical expectation $E(X) = \Sigma p_i x_i$

$$= 0 \times \frac{1}{4} + 1 \times \frac{2}{4} + 2 \times \frac{1}{4}$$

$$= 0 + \frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1$$

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Question 7.
The least non-negative remainder when $3^{50}$ is divided by 7 is
(a) 4
(b) 3
(c) 30

(d) 1
Solution:
(c) To find non-negative remainder, when $3^{50}$ divided by 7
3 = 3 (mod 7)
⇒ $3^2 = 2$ (mod 7)
⇒ $3^3 = 6$ (mod 7)
⇒ $3^3 = -1$ (mod 7)
⇒ $(3^3)^{16} = (-1)^{16}$(mod 7)
⇒ $3^{48} \times 3^2 = 3^2$ (mod 7)
⇒ $3^{50} = 9$ (mod 7)
⇒ 2 (mod 7)
which is equivalent to 30 (mod 7),
∴ Option (c) is correct.

Question 8.
If the cash equivalent of a perpetuity of ₹300 payable at the end of each quarter is ₹ 24000, then rate of interest converted quarterly is
(a) 5%
(b) 4%
(c) 3%
(d) 2%
Solution:
(a) Let the rate of interest be r% per annum, then

$i = \frac{r}{400}$ (interest quarterly)

Here, $P = 24000, R = 300$

$$P = \frac{R}{i}$$

$$\Rightarrow \quad 24000 = \frac{300}{\frac{r}{400}}$$

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$$\Rightarrow \quad r = \frac{300 \times 400}{2000} = 5\%$$

Question 9.
The value of ∫logxx dx is
(a) logx2 + C
(b) (logx)2x + C
(c) log x + C
(d) None of these

Solution:

(b) Let $I = \int \frac{\log x}{x} dx$

On putting $\log x = t$

$\Rightarrow \frac{1}{x} dx = dt$ LearnCBSE.in

$\therefore \quad I = \int t dt = \frac{t^2}{2} + C = \frac{(\log x)^2}{2} + C$

Question 10.
The supply of finished good was delayed for a month due to landslide in hilly terrain. Under which trend oscillation does this situation fall
(a) Seasonal
(b) Cyclical
(c) Secular
(d) Irregular
Solution:
(d) The supply of finished good was delayed for a month due to landslide in hilly terrain is irregular trend.

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Question 11.
A machine costing ₹30000 is expected to have a useful life of 4 yr and a final scrap value of ₹4000. Then, the annual depreciation is
(a) ₹5500
(b) ₹6500
(c) ₹7500
(d) ₹8500
Solution:
(b) Here, original cost of machine = ₹ 30000
Scrap value of machine = ₹ 4000 Useful life = 4yr
$\therefore$ Annual depreciation

$$= \frac{\text{Original cost} - \text{Scrap value}}{\text{Useful life}}$$

$$= \frac{30000 - 4000}{4}$$

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$$= \frac{26000}{4} = 6500$$

Hence, depreciation = ₹ 6500

Question 12.
The effective rate of interest equivalent to the nominal rate 6% compounded semi-annually is
(a) 6.05%
(b) 6.07%
(c) 6.09%
(d) None of these

Solution:

(c) $r_{eff} = \left[\left(1+\frac{r}{m}\right)^m - 1\right] \times 100$

Here, $r = 6\%$, $m = 2$ LearnCBSE.in

$r_{eff} = \left[\left(1+\frac{6}{200}\right)^2 - 1\right] \times 100$

$= [(1.03)^2 - 1] \times 100 = (1.609 - 1) \times 100$

$= 0.609 \times 100 = 6.09\%$

Question 13.
If the investment of ₹20000 in the mutual – fund in 2015 increased to ₹32000 in year 2020, then CAGR (Compound Annual Growth Rate is) is [given $(1.6)^{1/5}$ = 1.098]
(a) 9.08%
(b) 9.8%
(c) 0.098%
(d) 0.09%
Solution:
(b) We know,

$$CAGR = \left[\left(\frac{\text{Final value}}{\text{Beginning value}}\right)^{\frac{1}{n}} - 1\right] \times 100$$

Here, FV = ₹ 32000
BV = ₹ 20000 LearnCBSE.in
$n = 50$

$\therefore$ $CAGR = \left[\left(\frac{32000}{20000}\right)^{\frac{1}{5}} - 1\right] \times 100 = ((1.6)^{1/5} - 1)$

$= (1.098 - 1) \times 100 = 9.8\%$

Question 14.
The integrating factor of the differential equation $x\frac{dy}{dx} + 2y = x^3$ ($x \neq 0$) is ax
(a) x
(b) log x
(c) $x^2$
(d) $\frac{1}{x^2}$

Solution:

(c) Given, $x\frac{dy}{dx} + 2y = x^3$

$\Rightarrow \quad \frac{dy}{dx} + \frac{2y}{x} = x^2$

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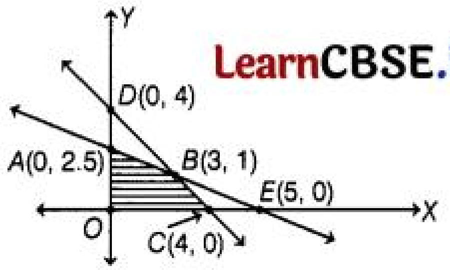
$\text{IF} = e^{\int \frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$

Question 15.

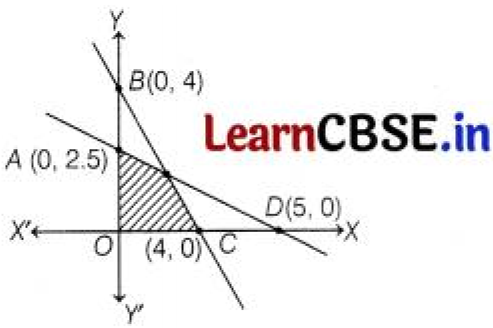
Besides non negativity constraint the figure given below is subject to which of the following constraints




(a) x + 2y ≤ 5 ; x + y ≤ 4
(b) x + 2y ≥ 5 ; x + y ≤ 4
(c) x + 2y ≥ 5 ; x + y ≥ 4
(d) x + 2y ≤ 5 ; x + y ≥ 4

Solution:

(a) Equation of BC




$$\frac{x}{4} + \frac{y}{4} = 1$$

$$x + y = 4$$

$$\therefore \quad x + y \le 4$$

Equation of $AD$

$$\frac{x}{5} + \frac{y}{2.5} = 1$$

$$x + 2y = 5 \Rightarrow x + 2y \le 5$$

∴ Constraints x + y ≤ 4, x + 2y ≤ 5

Question 16.

If X is a Poisson variate such that 3P(X = 2) = 2P(X = 1), then the mean of the distribution is equal to

(a) 43
(b) 34
(c) –43
(d) –34
Solution:
(a) Here, X is a Poisson distribution
Given, 3P(X = 2) = 2P(X = 1)

$$\Rightarrow \quad \frac{3\lambda^2 e^{-\lambda}}{2!} = \frac{2\lambda^1 e^{-\lambda}}{1!}$$

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[∵ λ = mean of distribution]

$$\Rightarrow \quad 3\lambda = 4$$

⇒ λ = 43
∴ Mean distribution is 43
Question 17.
For the given five values 35, 70, 36, 59, 64, the three years moving averages are given by
(a) 47, 53, 55
(b) 53, 47, 45
(c) 47, 55, 53
(d) 45, 55, 57
Solution:
(c) Given, 35, 70, 36, 59, 64
Three moving average is

$$\frac{35+70+36}{3}, \frac{70+36+59}{3}, \frac{36+59+64}{3}$$
$$= \frac{141}{3}, \frac{165}{3}, \frac{159}{3} = 47, 55, 53$$

Question 18.
The data point of a normal variate with mean 12, standard deviation 4 andZ-score 5 is
(a) 28
(b) 304
(c) 34
(d) 32
Solution:
(d) Here, μ = 12 , σ = 4 and Z = 5
z = x−μσ ⇒ o = x−124
⇒ 20 = x – 12 ⇒ x = 32

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Directions In the question number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Question 19.
Assertion (A) The maximum profit that a company makes if profit function is given by P(x) = 41 + 24x – 8x²; where 'x is the number of units and P is the profit in rupees is 59.
Reason (R) The profit is maximum at x-a ifP'(tz) = 0 andP"(a)>0
Solution:
(c) Here, P = 41 + 24x – 8x²

$$\frac{dP}{dx} = 24 - 16x$$

$$\frac{d^2P}{dx^2} = -16 < 0$$

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For maxima $\frac{dP}{dx} = 0$

$$\Rightarrow \quad 24 - 16x = 0 \Rightarrow \quad x = \frac{24}{16} = \frac{3}{2}$$

$\therefore \quad x = \frac{3}{2}$ is point of maxima

Max profit at $x = \frac{3}{2}$ is

$$P\left(\frac{3}{2}\right) = 41 + 24 \times \frac{3}{2} - 18 \times \left(\frac{3}{2}\right)^2$$

$$41 + 36 - 18 = 59$$

∴ Assertion (A) is true. For maxima
P'(x) = 0 and P"(x) < 0
∴ Reason (R) is false.

Question 20.
Assertion (A) The probability of getting 6 heads when a unbiased coin is tossed 10 times is C(10, 6) $(12)^{10}$
Reason (R) In a Binomial distribution the probability is given by P(X = r) = C(n, r)$(P^r(q)^{n-r}$.
Solution:

(a) Here, $n = 10, r = 6, P = \frac{1}{2}, q = \frac{1}{2}$

$$\therefore \quad P(X = 6) = {}^{10}C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4 = {}^{10}C_6\left(\frac{1}{2}\right)^{10}$$

∴ Assertion (A) and Reason (R) both are true and Reason (R) is correct explanation of Assertion (A).

## Section B

All questions are compulsory. In case of internal choice, attempt any one question only

Question 21.
At what rate of interest will the present value of perpetuity of ₹1500 payable at the end of every 6 months be ₹20,000?
Solution:
Let the rate of interest be r% per annum
Here, rate of interest is half-yearly

∴ i = r200
Given, R = ₹ 1500 and P = ₹ 20000

$$\therefore \quad P = \frac{R}{i}$$

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$$= 20000 = \frac{1500}{\frac{r}{200}} = r = \frac{1500 \times 200}{20000} = 15\%$$

∴ Rate interest = 15% per annum
Question 22.

At $A$ is a square matrix $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ such that $A^2 = pA$, then find the value of $p$.

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If $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew-symmetric matrix,
then find value of a + b + c.
Solution:

$$\text{Given,} \quad A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4+4 & -4-4 \\ -4-4 & 4+4 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

$$A^2 = 4\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 4A$$

$$\therefore \quad P = 4$$

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Or

Here, $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric

∴ a = -2, b = 0 and c = -3
Hence, a + b + c = -2 + 0 – 3 = -5

Question 23.
A Cooperative Society of farmers has 10 hectares of land to grow two crops A and B. To control weeds, pesticide has to be used for crops A and B at the rate of 30 g and 15 g per hectare. Further, no more than 750 g of pesticide should be used. The profit from crops A and B per hectare are estimated as ₹8000 and ₹9500. Formulate the above problem as LPP, in order to allocate land to each crop for maximum use.
Solution:
Let 'x' hectare and 'y' hectare of land be allocated to crop A and crop B.

According to the problem,
Maximise Z = 8000x + 9500y
Subject to constraints
$x + y \leq 10$
$30x + 15y \leq 750 \Rightarrow 2x + y \leq 50$
$\therefore x, y \geq 0$

Question 24.
A boatman takes twice as long as to go upstream to a point as to return downstream to the starting point. If the speed of a boat in still water is 15 km/hr what is the speed of the stream.
Or
'A can run 40 m while 'B' runs 50 m in the same time. In a 1000 m race, find by how much distance 'B' beats 'A ?
Solution:
Let the speed of 3 stream = x km/h
and speed of boat in still water = 15 km/h
Speed of boat in upstream = (15 – x) km/h
Speed of boat in downstream = (15+ x) km/h

Given, $\frac{\text{Time taken upstream}}{\text{Time taken downstream}} = \frac{2}{1}$

$$\frac{\frac{d}{15-x}}{\frac{d}{15+x}} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{15+x}{15-x} = \frac{2}{1}$$

$$\Rightarrow \quad 15 + x = 30 - 2x$$

$$\Rightarrow \quad 3x = 15$$

$$\Rightarrow \quad x = 5$$

Speed of stream = 5 km/h

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When $B$ runs 50 m, $A$ runs 40 m

When $B$ runs 1 m, $A$ runs $\frac{40}{50} = \frac{4}{5}$ m

When $B$ runs 1000 m, $A$ runs $= \frac{4}{5} \times 1000 = 800$ m

Hence, 6 beats A by 200 m

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Question 25.
A machine produces washers of thickness 0.50 mm. To determine whether the machine is in proper working order, a sample of 10 washers is chosen for which the mean thickness is 0.53 mm and the standard deviation is 0.03 mm. Test the hypothesis at 5% level of significance that the machine is working in proper order.

[Given $t_{0.025}$ = 2.262 at 9 degree of freedom]
Solution:
Null hypothesis $H_0$ alternate hypothesis $H_1$ as follows
$H_0 = \mu = 0.50$ mm
$H_1 = \mu \neq 0.50$ mm
Thus, a two tailed test is applied under hypothesis H0, we have

$$t = \frac{\bar{x} - \mu}{s}\sqrt{n-1}$$

Here, $\bar{x} = 0.53$

$\mu = 0.50$

$s = 0.03$

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$n = 10$

$$\therefore \quad t = \frac{0.53 - 0.50}{0.03} \times \sqrt{9} = \frac{0.03}{0.03} \times 3 = 3$$

Since, the calculated value of t = 3
> $t_{0.025}$ = 2.262,
The null hypothesis H0 can be rejected.
Hence, we conclude that machine is not working properly.

## Section C

All Questions are compulsory. In case of internal choice, attempt any one questions only

Question 26.
Evaluate $\int$x3x+2dx
Or
Evaluate $\int (x^2 + 1) \log_e x \, dx$

Solution:

Let $I = \int \frac{x^3}{x+2} dx$

$$I = \int \left( x^2 - 2x + 4 - \frac{8}{x+2} \right) dx$$

$$I = \frac{x^3}{3} - \frac{2x^2}{2} + 4x - 8 \log (x+2) + C$$

$$\Rightarrow I = \frac{x^3}{3} - x^2 + 4x - 8 \log (x+2) + C$$

*Or*

Let $I = \int (x^2 + 1) \log x \, dx$

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Integrating by parts

$$I = \log x \int (x^2 + 1) dx - \int \left\{ \left( \frac{d}{dx} \log x \right) \cdot \int (x^2 + 1) dx \right\} dx + C$$

$$= \log x \left( \frac{x^3}{3} + x \right) - \int \frac{1}{x} \left( \frac{x^3}{3} + x \right) dx + C$$

$$= \log x \left( \frac{x^3}{3} + x \right) - \int \left( \frac{x^2}{3} + 1 \right) dx + C$$

$$= \log x \left( \frac{x^3}{3} + x \right) - \left( \frac{x^3}{9} + x \right) + C$$

Question 27.
Cost of two toys A and B are ₹50 and ₹75. On a particular Sunday, shopkeeper P sells 7 toys of type A and 10 toys of type B whereas shopkeeper Q sells 8 toys of type A and 6 toys of type B. Find income of both shopkeepers' using matrix Algebra.
Solution:
Given, Cost of toys A = ₹50
Cost of toys B = ₹75
P sells 7 toys of type A and 10 toys of type B
Q sells 8 toys of type A and 6 toys of type B

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$$\begin{matrix} & A & B \\ P & 7 & 10 \\ Q & 8 & 6 \end{matrix} \quad \begin{bmatrix} 7 & 10 \\ 8 & 6 \end{bmatrix} \begin{bmatrix} 50 \\ 75 \end{bmatrix} = \begin{bmatrix} 350 + 750 \\ 400 + 450 \end{bmatrix} = \begin{bmatrix} 1100 \\ 850 \end{bmatrix}$$

Income of shopkeeper P = ₹ 1100
and income of shopkeeper Q = ₹ 850

Question 28.
Find the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x - 5$ is increasing or decreasing.
Solution:
Given, $f(x) = 2x^3 - 9x^2 + 12x - 5$
$f'(x) = 6x^2 - 18x + 12$

f'(x) = 6 ($x^2$ – 3x + 2) = 6(x – 2)(x – 1)
On putting f'(x) = 0
6 (x – 2) (x – 1) = 0
x = 2, 1 are the critical points.
The intervals are (-∞ 1), (1, 2), (2, ∞).

| Intervals | Sign of f'(x) | Nature of f(x) |
|---|---|---|
| (-∞, 1) | (+) (-) (-) > 0 | ↑ increasing |
| (1, 2) | (+) (-) (+) < 0 | ↓ decreasing |
| (2, ∞) | (+) (+) (+) > 0 | ↑ increasing |

Increasing x ∈ (-∞, 1) ∪ (2, ∞)
Decreasing x ∈ (1, 2)

Question 29.
The demand and supply functions under the pure market competition are $p_d = 16 – x^2$ and $p_s = 2x^2 + 4$ respectively, where p is the price and x is the quantity of the commodity. Using integrals find Consumer's surplus.
Or
The demand and supply functions under the pure market competition are $p_d = 56 – x^2$ and $p_s$ = 8 + x23 respectively, where p is the price and x is the quantity of the commodity. Using integrals find Producer's surplus.
Solution:
Given, $P_d$ = 16 – x⇒ and $P_s = 2x^2 + 4$
Under pure competition $P_d = P_s$
⇒ 16 – x⇒ = 2x ⇒ + 4
⇒ 3x⇒ = 12
x = 2, -2
x = 2, x ≠ -2
When, $x_0 = 2$, $P_0$ = 16 – 4 = 12

Hence, consumer surplus $= \int_0^{x_0} P_d\,dx - P_0\,x_0$

$$= \int_0^2 (16 - x^2)\,dx - 12\,x^2$$

$$= \left[16x - \frac{x^3}{3}\right]_0^2 - 24$$

$$= 32 - \frac{8}{3} - 24$$

$$= \frac{16}{3} \text{ units}$$

*Or*

Here, $P_d = 56 - x^2$ and $P_s = 8 + \dfrac{x^2}{3}$

Under pure competition

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$$P_d = P_s$$

$$\Rightarrow \quad 56 - x^2 = 8 + \frac{x^3}{3}$$

$$\Rightarrow \quad \frac{x^2}{3} + x^2 = 56 - 8$$

$$\Rightarrow \quad \frac{4x^2}{3} = 48$$

$$\Rightarrow \quad x^2 = 36$$

$$\Rightarrow \quad x = \pm 6$$

$$\Rightarrow \quad x = 6, \quad x \neq 6$$

When $x_0 = 6$, then $P_0 = 20$

Hence, produces surplus, $P_0 x_0 - \int_0^{x_0} P_s\,dx$

$$20 \times 6 - \int_0^6 \left(8 + \frac{x^2}{3}\right) dx$$

$$= 120 - \left[8x + \frac{x^3}{9}\right]_0^6$$

$$= 120 - \left(48 + \frac{216}{9}\right)$$

$$= 120 - (48 + 24)$$

$$= 120 - 72$$

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$$= 48 \text{ units}$$

Question 30.

Mr Surya borrows a sum of ₹500000 with total interest paid ₹200000(flat) and he is paying an EMI of ₹12500. Calculate loan tenure.

Solution:
Here, P = ₹ 500000
Interest (I) = 200000 (flat), EMI = 12500
For flat tenure,

$$\text{EMI} = \frac{P + I}{n}$$

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$$12500 = \frac{500000 + 200000}{n} = \frac{700000}{n}$$

$$n = \frac{700000}{12500} = 56 \text{ months}$$

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Question 31.
Mr Sharma wants to send his daughter abroad for higher studies after 10 yr. He sets up a sinking fund in order to have ₹500000 after 10 yr. How much should he set aside biannually into an account paying 5% per annum compounded annually.
[use $(1.025)^{20}$ = 1.6386]
Solution:
Given, S = ₹ 500000,

$r$ = 5% compounded half yearly

$$i = \frac{5}{200} = 0.025$$

$$n = 10 \times 2 = 20$$

$$R = \frac{i \times S}{(1+i)^n - 1}$$

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$$\Rightarrow \quad R = \frac{0.025 \times 500000}{(1 + 0.025)^{20} - 1}$$

$$\Rightarrow \quad R = \frac{12500}{1.6386 - 1} \quad [\because (1.025)^{20} = 1.6386]$$

$$\Rightarrow \quad R = \frac{12500}{0.6386} = 19574.07$$

Thus, ₹ 19574.07 deposited half-yearly in sinking fund.

## Section D

All questions are compulsory. In case of internal choice, attempt any one questions only

Question 32.
On doing the proof reading of a book on an average 4 errors in 10 pages were detected. Using Poisson's distribution find the probability of (i) No error and (ii) one error fn 1000 pages of first printed edition of the book, (given $e^{-0.4}$ = 0.6703)
Or
How many time must Sunil toss a fair coin so that the probability of getting at least one head is more than 90%?

Solution:

Here, mean $(\lambda) = \frac{4}{10} = 0.4$

For Poisson distribution

$$P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-0.4}(0.4)^r}{r!}$$

$$= \frac{(0.6703)\ (0.4)^r}{r!}$$

$$[\because\ e^{-0.4} = 0.6703]$$

In 1000 pages error $= \frac{1000 \times 0.6703\ (0.4)^r}{r!}$

(i) For zero error

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$$P(X = 0) = \frac{1000 \times 0.6703\ (0.4)^0}{0!} = 670.3$$

(ii) For one error

$$P(X = 1) = \frac{670.3 \times 0.4^1}{1!} = 670.3 \times 0.4$$

$$= 268.12$$

Or

Let a coin B tossed n time.

Probability of Head $(P) = \frac{1}{2}$

$$q = 1 - P = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(X = r) = {}^nC_r\ p^r q^{n-r}$$

$$P(X = 0) = {}^nC_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^n$$

Given, $P(X \geq 1) > 90\% = \frac{90}{100} = \frac{9}{10}$

$$P(X \geq 1) = 1 - P\ (X = 0)$$

$$1 - \left(\frac{1}{2}\right)^n > \frac{9}{10}$$

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$$\left(\frac{1}{2}\right)^n < 1 - \frac{9}{10} = \frac{1}{10}$$

$$2^n > 10 \Rightarrow n > 4$$

⇒ n > 4 n is 4 or more times.

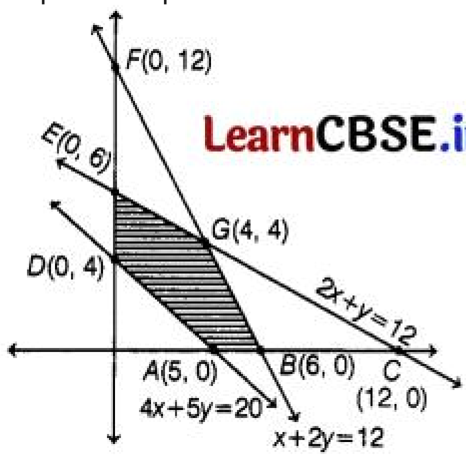
Question 33.

A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 h, whereas machine III must be operated for at least 5 h a day. He produces only two items M and N, each requiring the use of all the three machines.

The number of hours required for producing 1 unit of M and N on three machines are given in the following table:

| Items | Number of hours required on machines | | |
|---|---|---|---|
| | I | II | III |
| M | 1 | 2 | 1 |
| N | 2 | 1 | 1.25 |

He makes a profit of ₹600 and ₹400 on one unit of items M and N respectively. How many units of each item be produced, so as to maximise the profit. What is the maximum profit?
Solution:
Let x and y be the number of units of items M and N, respectively.
Maximise Z = 600x + 400y
Subject to constraints
$x + 2y \leq 12$
$2x + y \leq 12$
$x + 125y \leq 5 \Rightarrow 4x + 5y \leq 20$
$x, y \geq o$
Graph of inequalities




On solving 2x + y = 12 and x + 2y = 12, we get
Point G (4, 4)
Corner points are A(5, 0), B(6, 0), G(4, 4), E(0, 6) D(0, 4)

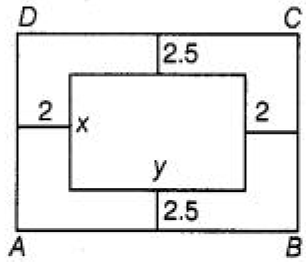
| Comer points | Z = 600x + 400y |
|---|---|
| A(5, 0) | 3000 |

| | |
|---|---|
| 6(6, 0) | 3600 |
| G(4, 4) | 4000 (Maximum) |
| E(0, 6) | 2400 |
| D(0, 4) | 1600 |

Hence, maximum profit is ₹4000 at (4, 4) i.e. 4 units of each the items M and N.

Question 34.
A company produces a certain commodity with ₹2400 fixed cost. The variable cost is estimated to be 25% of the total revenue received on selling the product at a rate of ₹8 per unit. Find the following
(i) Cost Function.
(ii) Revenue Function
(iii) Break-even Point
(iv) Profit Function
Or
The production manager of a company plans to include 180 sq cm. of actual printed matter in each page of a book under production. Each page should have a 2.5 cm wide margin along the top and bottom and 2 cm wide margin along the sides. What are the most economical dimensions of each printed page?
Solution:
Let x unit of product be produced and sold. As, selling price of one unit is ₹8.
Total revenues on x units i.e. $R(x) = 8x$
(i) Cost function
C(x) = Fixed cost + 25% of total revenue 2S
$= 2400 + \frac{25}{100} \times 8x$
$= 2400 + 2x$
(ii) Revenue function $R(x) = 8x$
(iii) Break-even point, when $R(x) = C(x)$
$8x = 2400 + 2x$
$\Rightarrow 6x = 2400$
$\Rightarrow x = \frac{2400}{6} = 400$
Break-even point at $x = 400$
(iv) Profit function $= R(x) – C(x) = 8x – 2400 – 2x$
$= 6x – 2400$
Or
Let x and y be dimension of printed pages, then

$$xy = 180$$




Area of page $A = (x + 5)(y + 4)$

$$= xy + 5y + 4x + 20$$

$$= 180 + 4x + 5\left(\frac{180}{x}\right) + 20$$

$$= 200 + 4x + \frac{900}{x}$$

$$\frac{dA}{dx} = 4 - \frac{900}{x^2}$$

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For most economical dimension $\frac{dA}{dx} = 0$

$$4 = \frac{900}{x^2} = 0$$

$\Rightarrow$ $x^2 = 225$

$\Rightarrow$ $x = 15$

Now, $\frac{d^2A}{dx^2} = \frac{1800}{x^3} > 0$

$\therefore$ $y = \frac{180}{15} = 12$ [from Eq. (i)]

$$\left(\frac{d^2A}{dx^2}\right)_{x=15} = \frac{1800}{(15)^3} > 0$$

$\therefore$ A is minimum.
Hence, most economical dimension are
x + 5 = 15 + 5 = 20 cm = length
and y + 4 = 12 + 4 = 16cm = width

Question 35.
The management committee of a Welfare Club decided to award some of its members (say x) for sincerity, some (say y) for helping others selflessly and some others (say z) for effective management. The sum of all the awardees is 12. Three times the sum of all awardees for helping others selflessly and effective management added to two times the number of awardees for sincerity is 33. If the sum of the number of awardees for sincerity and effective management is twice the number of awardees for helping others, use matrix method to find the number of awardees of each category.
Solution:
Given, sincerity = x Helping others selflessly = y

Some others = z
According to the problem,
x + y + z = 12
2x + 3y + 3z = 33
x – 2y + z = 0
In matrix form

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

Here, $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}$

$$|A| = 1(3+6) - 1(2-3) + 1(-4-3)$$
$$= 9 + 1 - 7 = 3$$
$$|A| \neq 0$$

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$\therefore$ $A^{-1}$ exists.

$$\text{adj } A = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

and $A^{-1} = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$

$$AX = B$$
$$\Rightarrow \quad X = A^{-1}B$$

$$\therefore X = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 3 \end{bmatrix}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 108 - 99 + 0 \\ 12 + 0 - 0 \\ -84 + 99 + 0 \end{bmatrix}$$

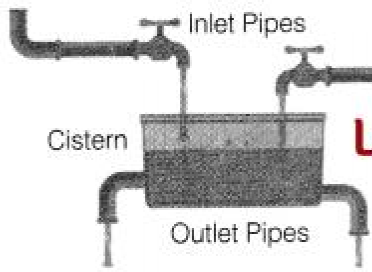
$$X = \frac{1}{3}\begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

Hence, x = 3, y = 4 and z = 5
Awardees for sincerity = helping other selflessly and some others are 3, 4, 5 respectively.

## Section E

AH questions are compulsory. In case of internal choice, attempt any one questions only

Question 36.
A, B and C are three pipes connected to a tank. A and B together fill the tank in 6 h. B and C together fill the tank in 10 h. A and C together fill the tank in 712 h.
Based on above information answer the following questions.

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(i) In how much time will A, B and C fill the tank?
(ii) In how much time will A separately fill the tank?
(iii) In how much time will B separately fill the tank?
Or
In how much time will C separately fill the tank?
Solution:
Given, A and B fill the tank in 6 h
B and C fill the tank in 10 h
A and C fill the tank in $7\frac{1}{2}$h = $\frac{15}{2}$h
(i) A, B and C fill the tank

$$=2\left(\frac{6\times 10\times \frac{15}{2}}{6\times 10+6\times \frac{15}{2}\times 10\times \frac{15}{2}}\right)\text{h}$$

$$=2\left[\frac{450}{60+45+75}\right]$$

$$=\frac{2\times 225}{180}=5\text{h}$$

(ii) *A* separately fill the tank

$$=((A+B+C)-(B+C))\text{ fill the tank}$$

$$=\frac{1}{\frac{1}{5}-\frac{1}{10}}=\frac{10\times 5}{10-5}$$

$$=\frac{50}{5}=10\text{h}$$

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(iii) *B* separately fill the tank

$$=((A+B+C)-(A+C))\text{ fill the tank}$$

$$=\frac{1}{\frac{1}{5}-\frac{2}{15}}=\frac{15}{3-2}=15\text{h}$$

*Or*

*C* separately fill the tank

$$=((A+B+C)-(A+B))\text{ fill the tank}$$

$$=\frac{1}{\frac{1}{5}-\frac{1}{6}}=\frac{6\times 5}{6-5}=30\text{ h}$$

Question 37.
Let X denote the number of hours a person watches television during a randomly selected day. The probability that X can take the values $x_i$, has the following form, where 'k' is some unknown constant.

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$$P(X = x_i) = \begin{cases} 0.2, & \text{if } x_i = 0 \\ kx_i, & \text{if } x_i = 1 \text{ or } 2 \\ k(5 - x_i), & \text{if } x_i = 3 \\ 0, & \text{otherwise} \end{cases}$$

Based on above information answer the following questions.
(i) Find the value of k.
(ii) What is the probability that a person watches two hours of television on a selected day?
(iii) What is the probability that the person watches at least two hours of television on a selected day?
Or
What is the probability that the person watches at most two hours of television on a selected day?
Solution:

From given information, the probability distributions of X is

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(X) | 0.2 | $k$ | $2k$ | $2k$ |

(i) We know, $\Sigma P_1 = 1$

$\therefore \quad 0.2 + k + 2k + 2k = 1$

$$= 5k = 1 - 0.2 = 0.8$$

$$= k = \frac{0.8}{5} = \frac{8}{50} = \frac{4}{25}$$

(ii) $P(X = 2) = 2k = 2 \times \frac{4}{25} = \frac{8}{25}$

(iii) $P(X \geq 2) = P(X = 2) + P(X = 3)$

$$= 2k + 2k = 4k = 4 \times \frac{4}{25} = \frac{16}{25}$$

*Or*

$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= 0.2 + k + 2k = 0.2 + 3k$$

$$= 0.2 + \frac{3 \times 4}{25}$$

$$= \frac{2}{10} + \frac{12}{25}$$

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$$= \frac{1}{5} + \frac{12}{25}$$

$$= \frac{5 + 12}{25}$$

$$= \frac{17}{25}$$

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Question 38.
When observed over a long period of time, a time series data can predict trend that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or prediction of future estimated sales or production.
The table below shows the welfare expenses (in lakh ₹) of Steel Industry during 2001-2005. Fit a straight line trend by the method of least squares and estimate the trend for the year 2008.
Determine the trend of rainfall by three years average and draw the moving averages graph.
Solution:
Given, table

| Year | $y$ | $x = t_i - 2003$ | $x^2$ | $xy$ |
|---|---|---|---|---|
| 2001 | 160 | –2 | 4 | –320 |
| 2002 | 185 | –1 | 1 | –185 |
| 2003 | 220 | 0 | 0 | 0 |
| 2004 | 300 | 1 | 1 | 300 |
| 2005 | 510 | 2 | 4 | 1020 |
| | $\Sigma y = 1375$ | $\Sigma x = 0$ | $\Sigma x^2 = 10$ | $\Sigma xy = 815$ |

$$\therefore \quad a = \frac{\Sigma y}{n} = \frac{1375}{5} \qquad [\because n = 5]$$

$$\Rightarrow \quad a = 275$$

$$\therefore \quad b = \frac{\Sigma xy}{\Sigma x^2} = \frac{815}{10} = 81.5$$

So, the required equation of the straight line trend is $y_t = a + bx$
$\Rightarrow y_t = 275 + 81.5x$
Now, for calculated yf for the year 2008, x will be 5.
For year 2008, x = 5
$\because y_t = 275 + (81.5)\,5$
$= 275 + 407.5$

= 682.5

*Or*

Given, table

| Year | Rainfall (in cm) | 3 yr moving total | 3 yr moving average |
|---|---|---|---|
| 2001 | 1.2 | | |
| 2002 | 1.9 | 5.1 | 1.70 |
| 2003 | 2 | 5.3 | 1.77 |
| 2004 | 1.4 | 5.5 | 1.83 |
| 2005 | 2.1 | 4.8 | 1.60 |
| 2006 | 1.3 | 5.2 | 1.73 |
| 2007 | 1.8 | 4.2 | 1.40 |
| 2008 | 1.1 | 4.2 | 1.40 |
| 2009 | 1.3 | | |

Graph of above data

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